CRITERIA FOR CEILING SLOT–VENTILATED AGRICULTURAL ENCLOSURES: NON–ISOTHERMAL

H. Yu, S. J. Hoff

ABSTRACT. Experiments using two scale–models were conducted to study non–isothermal airflow similarity in a ceiling slot–ventilated agricultural enclosure. The main criteria used for determining similarity were airflow pattern similarity, occupied zone airspeed, and temperature similarity. For agricultural ventilation issues, these were projected to be paramount to a successful scale–model study. The current study, based on past agricultural ventilation studies, focused attention on the Archimedes number (Ar) and Momentum ratio (Rm) as the similitude criteria for non–isothermal airflow. Results indicated that Ar is the appropriate similitude criteria for non–isothermal airflow patterns under strong buoyancy–affected conditions where Ar > 0.015. With increasing airflow or decreasing temperature differences, resulting in a lower buoyancy–affected air–jet, Rm with a consistent overall temperature differential was determined to be the appropriate similitude criteria. This study focused on practical considerations for conducting scale model studies such as scale–model conditions measured in the animal occupied zone and its relation to expected prototype behavior.

Keywords. Non–isothermal, Archimedes number, Momentum ratio, Airflow, Slot–ventilated.

Muljeans (1966) applied the similarity principle to the differential equations governing non–isothermal airflow and found parameters governing similarity between a scale–model and prototype. He showed that the Reynolds (Re), Archimedes (Ar), and Peclet (Pe) numbers were the governing dimensionless parameters. Neglecting the effects of molecular friction on airflow and molecular heat transfer on energy exchange, a simplified scaling model was derived using the Ar. He used three geometrically similar rooms with linear dimensions in the ratio of 1:3:9 and provided non–isothermal conditions using heated walls. The experimental results showed that airflow patterns were clearly dominated by Ar and independent of Re, even for low–turbulence conditions (i.e., Re < 100). The airflow approximated that of isothermal airflow at and below a critical Ar value. The critical Ar varied with the geometric scale and was approximated as \( Ar(bh/D^2) \leq 40 \). Fully turbulent airflow was found for \( \text{Inlet } Ar > 1500 \).

Baturin (1972) studied scale–model theory and the rules associated with distorted models for industrial ventilation and variations in fluid density. Fluid densities were altered using either non–isothermal flow or by using different fluids in isothermal flow. He showed that the scaling criteria should be based on Ar provided that the Re is in the turbulent fluid region.

Yao et al. (1986) studied a neutral–pressure ventilated swine barn with a 1:12 scale–model and summer ventilation data in low turbulent airflow. The effects of internal temperature difference and obstructions were compared. It was concluded that Ar was maintained as the undistorted similitude parameter and Re was neglected as a distorted parameter, since air movement was believed relatively independent of Re within the enclosures, but Re became important at higher airflow. Christianson et al. (1988) validated the above similitude modeling approach by comparing the pig–level air velocity between the 1:12 scale–model and prototype. The scale–model over predicted velocity by approximately 3 times when based on Ar and underestimated it by 17 times when based on Re. They concluded that Ar is more important than Re in low velocity flow conditions for non–isothermal airflow.

Fissore and Liebecq (1991) derived the parameters for predicting velocity distribution and temperature gradients using a 1:3 scale–model. They studied the effects of Ar, inlet velocity, slot inlet length, and building length. They developed a simple empirical model using Ar for cases where Re remained above a critical value of 1850 (Fissore and Liebecq, 1990). The results were validated using measurements of temperature and velocity in the prototype and scale–model.

Zhang et al. (1993) conducted similitude modeling for predicting room air motion for non–isothermal airflow. They investigated similitude modeling using both theoretical and experimental techniques using a prototype and a 1:4 scale–model. A new scaling method was derived based on the relative deviation of Ar from a critical Ar. The critical Ar was that at which the air–jet fell immediately after entering the building. They found that the critical Ar decreased as the scale–model size decreased. The new scaling criterion was a compromise between the Ar and Re. The results showed good agreement between scale–model and prototype. The model
slightly over-predicted mean air velocity and under-predicted turbulent intensity and turbulent kinetic energy. A compensation coefficient accounting for the effect of temperature difference was suggested to improve the scaling model.

Practical implications of buoyancy-affected air-jets have been studied in livestock ventilation systems. Zhang et al. (1996) developed guidelines on buoyancy-affected air-jets and inlet control strategies for controlling air-jet trajectories. Vranken and Berckmans (1997) proposed an inlet controller based on the air-jet Ar.

**OBJECTIVES**

There is a need to clarify similitude criteria of airflow between a scale-model and prototype ceiling slot-ventilated agricultural enclosure under non-isothermal conditions. The objective of this project was to test the use of Archimedes number (Ar) as an acceptable non-isothermal similitude criterion. The results will be helpful for developing guidelines when using scale-models to assess realistic prototype behavior in slot-ventilated livestock facilities.

**SIMILITUDE ANALYSIS**

The traditional method for investigating similitude requirements uses dimensional analysis and the Buckingham Pi theorem. Only knowledge of the variables related to the problem of interest is required. The risk with this method is that if one or more important variables are neglected, serious mistakes of scale-model design could result (Young, 1994).

A more sophisticated method of similitude analysis is derived from the governing differential equations and the associated initial and boundary conditions. This method gives the necessary Pi terms directly and is a more rigorous statement of similitude, but requires more knowledge and analysis of the problem (Shepherd, 1965). From this analysis, the following dimensionless parameters result:

**Froude number**

\[
Fr = \frac{U_c}{\sqrt{gL_c}}
\]

(1)

Fr represents the ratio of inertia to gravitational forces. Similarity requires that:

\[
\left( \frac{U_c}{\sqrt{gL_c}} \right)_m = \left( \frac{U_c}{\sqrt{gL_c}} \right)_p
\]

with \( \varepsilon_m = \varepsilon_p \), results in a relation between diffuser airspeed as:

\[
\frac{U_{c,m}}{U_{c,p}} = \sqrt{\frac{L_{c,m}}{L_{c,p}}} = \sqrt{n}
\]

**Archimedes number**

\[
Ar = \frac{\beta g L_c (T_i - T_d)}{U_c^2}
\]

(2)

where \( \beta = 1/T \) for perfect gases. The Ar represents the ratio of buoyant to inertial forces. Similarity requires that:

\[
\left( \frac{\beta g L_c (T_i - T_d)}{U_c^2} \right)_m = \left( \frac{\beta g L_c (T_i - T_d)}{U_c^2} \right)_p
\]

with \( \varepsilon_m = \varepsilon_p \), results in:

\[
\frac{\beta g L_c (T_i - T_d)}{U_c^2}_m = \frac{\beta g L_c (T_i - T_d)}{U_c^2}_p
\]

If the temperature fields are set equal, then \( \beta_m = \beta_p \) and \( (T_i - T_d)_m = (T_i - T_d)_p \), resulting in a relation between diffuser airspeed as:

\[
\frac{U_{c,m}}{U_{c,p}} \sqrt{\frac{L_{c,m}}{L_{c,p}}} = \sqrt{n}
\]

**Euler number**

\[
Eu = \frac{P_o - P_d}{\frac{1}{2} \rho U_c^2}
\]

(3)

Eu represents the ratio of pressure to momentum forces. Similarity requires:

\[
\left( \frac{P_o - P_d}{\frac{1}{2} \rho U_c^2} \right)_m = \left( \frac{P_o - P_d}{\frac{1}{2} \rho U_c^2} \right)_p
\]

If the same working fluid and temperature differentials exist between scale-model and prototype then \( \rho_m = \rho_p \) and the relation simplifies to:

\[
\left( \frac{P_o - P_d}{\frac{1}{2} \rho U_c^2} \right)_m = \left( \frac{P_o - P_d}{\frac{1}{2} \rho U_c^2} \right)_p
\]

If the pressure difference between inlet and outlet are the same between scale-model and prototype, the relation further simplifies to:

\[
U_{c,m} = U_{c,p}
\]

**Reynolds number**

\[
Re = \frac{\rho U_c L_c}{\mu}
\]

(4)

Re represents the ratio of inertia to viscous forces. Similarity between scale-model and prototype requires:

\[
\left( \frac{\rho U_c L_c}{\mu} \right)_m = \left( \frac{\rho U_c L_c}{\mu} \right)_p
\]

If the same working fluid between scale-model and prototype is used, then \( \rho_m = \rho_p \) and \( \mu_m = \mu_p \), resulting in the following requirement between diffuser airspeeds:

\[
\frac{U_{c,m}}{U_{c,p}} = \frac{L_{c,p}}{L_{c,m}} = n
\]
where \( n \) is the scale between prototype and scale–model.

**Peclet number**

\[
\text{Pe} = \frac{U_c L_c}{\alpha} 
\]

(5)

\( \text{Pe} \) represents dimensionless heat transfer and could also be expressed as \( \text{Re Pr} \). Similarity requires:

\[
\left( \frac{U_c L_c}{\alpha} \right)_m = \left( \frac{U_c L_c}{\alpha} \right)_p
\]

If the same working fluid between scale–model and prototype is used, then \( \alpha_m = \alpha_p \) resulting in the following requirement between diffuser airspeeds:

\[
\frac{U_{c,m}}{U_{c,p}} = \frac{L_{c,p}}{L_{c,m}} = n
\]

The scaling velocity is the same as that based on \( \text{Re} \). This result implies that the Prandtl number (\( \text{Pr} \)) is satisfied between the scale–model and prototype if the same working fluid is used.

**SCALING MODEL**

Complete similarity for non–isothermal airflow requires geometric similarity between the scale–model and prototype with similar boundary conditions and equal \( \text{Fr, Ar, Eu, Re, and Pe numbers} \). Only partial similarity is reached with scale–model studies because of the conflict between these similarity requirements. A distorted model is usually unavoidable. Using the predominant undistorted parameters as the similarity requirement is usually used to study regions of interest in the prototype (Zhang, 1991).

If air is used as the working fluid in both the scale–model and prototype, and the same thermal environment is maintained (\( T_f - T_d \)) then the same air properties such as \( \rho_m = \rho_p, v_m = v_p, \alpha_m = \alpha_p, \beta_m = \beta_p \) exist (Baturin, 1972). With consistent air properties, the Prandtl number will be satisfied and the governing similarity parameters become the \( \text{Eu, Re, Ar, and Fr numbers} \).

Based on the similarity parameters presented, \( \text{Re} \) requires higher inlet airspeed in the scale–model and \( \text{Eu} \) requires the same inlet airspeed in the scale–model. Conversely, \( \text{Ar} \) and \( \text{Fr} \) require lower inlet airspeeds in the scale–model. Reports suggest air movement in slot–ventilated enclosures with air–jet ventilation is affected mainly by thermal buoyancy, independent of \( \text{Re} \) when examined at higher ranges (Re>1850) (Baturin, 1972; Szucs, 1980; Yao et al., 1986; Christianson et al., 1988; Fissore and Liebecq, 1991) or lower (Re<1000) (Mullejans, 1966). For small \( \text{Ar} \), airflow approximates isothermal flow (Mullejans, 1966). The critical value of \( \text{Ar} \) depends on the geometric conditions between the scale–model and prototype (Zhang, 1991).

Randall and Battams (1979) postulated a corrected Archimedes number (\( \text{Ar}_c \)) that accounted for the properties of inlet aperture and room size to describe airflow patterns in livestock buildings. The air–jet remains horizontal as would an isothermal jet when \( \text{Ar}_c < 3.0 \), and falls after entry because of the dominant buoyant forces when \( \text{Ar}_c > 75 \).

Perfect thermal similarity is difficult to obtain because of the complex modes of heat transfer phenomena at the enclosure boundaries. However, satisfactory thermal similarity can be obtained between a scale–model and prototype by assuring geometric similarity, proportionately reducing the wall thickness and conductivity of the scale–model, and operating the heat input proportional to the square of the linear size reduction scale factor as (Parczewski and Renzi, 1963):

\[
\frac{q_p}{q_m} = n^2
\]

where \( q_p \) and \( q_m \) represent the total input heat rate for the prototype and scale–model, respectively. The factors \( q_p \) and \( q_m \) imply that the unit heat flux is equal between scale–model and prototype.

Pressure, inertial, buoyant, and viscous forces generally govern fluid motion in buoyancy–affected air–jets. The local characteristics of the flow are determined by the relative magnitude of these forces at each point. The overall characteristics are determined by the strength of the forces at the air–jet source and by ambient conditions (Chen and Rodi, 1980). Chen and Rodi (1980) concluded that for very small velocities or large temperature differences, \( \text{Ar} \) solely dominates the air–jet flow performance in a ventilated room. As the airflow rate increases or the temperature difference decreases, the airflow behaves like isothermal airflow. The critical value to distinguish the dominant parameters may depend on the specific structure of airflow conditions, such as enclosure dimensions, geometric configurations, and the amount of heat flux.

For isothermal airflow, the \( \text{Re} \) has traditionally been used as the scaling factor. However, in scale–model studies with confined wall jets where airflow pattern and air–jet penetration distance similarity have been measured, the momentum ratio (\( \text{Rm} = \frac{U^2_h}{h(L+H)} \)) has been proposed (Adre and Albright, 1994) and verified (Adre and Albright, 1994; Yu and Hoff, 1999) as a more appropriate scaling criterion. The \( \text{Rm} \) is functionally equivalent to the \( \text{Eu} \) number for similarity in diffuser airspeed between a scale model and prototype.

**MATERIALS AND METHOD**

**EXPERIMENTAL FACILITIES**

Two geometrically similar scale–models representing a 1:3 and 1:6 scale–model of a prototype swine–grower barn were used to study airflow parameters between a prototype (1:3) and scale–model (1:6). Airflow pattern, air–jet penetration distance, and variation in velocity and temperature fields were measured using airspeed/temperature measurements and airflow visualization.

The test chamber layout used for both physical models is shown in figures 1 and 2. The slot inlet width was the same as the enclosure width (W). As a result of the inlet aspect ratio being much larger than 20, the airflow was treated as a two–dimensional wall jet without the effect of side–walls (Forthmann, 1934). Dimensions of the two models are shown in figure 2. Both models were constructed from 12.7 mm thick (1/2 inch) plywood. The inner surface of the models were sanded and painted black. The front wall was made of Plexiglas to
accommodate airflow visualization. Access holes were placed on the top ceiling perpendicular to the inlet wall and the end wall at intervals of 20 mm, except for the area between the inlet and a distance of 33 h from the inlet where a continuous access slot was constructed to discretize air–jet development near the inlet. When not in use, all access holes were plugged with the inside surface level with the inside surface of the ceiling.

To accommodate airflow visualization, a portion of the top ceiling was fabricated with Plexiglas, which extended along the length of both models. A circular exhaust port was provided on the inlet wall as shown in figure 2. A 75 mm diameter hole was used for the 1:3 model, and a 150 mm diameter hole was used for the 1:3 model.

Ductwork was constructed and fitted between the circular exhaust hole and an exhaust fan. Calibrated orifice plates were used to select desired airflow rates through each model. A micromanometer (Model 1430, Dwyer Instruments, Inc.) was used to measure the pressure difference across the orifice to determine airflow rate. All surfaces of both models were insulated with 88.9 mm (3.5 inches) of fiberglass insulation to reduce heat conduction loss during each experiment. When visualizing airflow patterns, the insulation covering the Plexiglas walls was temporarily removed.

HEATING SYSTEM

The floor of both models was fitted with insulated silicone rubber (dark orange in color) heat panels (Model SRFG 442/2 and 1242/2, OMEGA Engineering, Inc.) that occupied 58.7 percent of the floor area to simulate animal surface temperature (fig. 3). The power flux was 0.0016 W/m² (2.5 W/in²) and was controlled by microprocessor–based temperature controllers (Model CN9000, OMEGA Engineering, Inc.) with an accuracy of ±0.5°C using a PID control module. The maximum operating temperature of the heaters was 120°C. The scale–model and prototype floors were maintained at a constant temperature to simulate the heat gains from animals in the occupied zone. The temperature differences (ΔT) between the top surface of the heated floor panel and the inlet air–jet were set at 10°C, 40°C, and 60°C to test non–isothermal air–jet behavior. All temperatures were measured with T–type thermocouples.

AIRFLOW PATTERN ASSESSMENT

Airflow patterns were visualized using titanium tetrachloride (Model 15–049, E. Vernon Hill, Inc.). The chemical tracer was introduced at the slot–inlet and allowed to entrain with the inlet air. A light box, illuminated by 300 Watt incandescent lamps, focused light in an approximate 5 cm wide light sheet along the longitudinal axis of each test chamber. The bright light combined with the black inner surfaces allowed for photographing of air movement by both camera and camcorder.

Airflow patterns were assessed using both airflow visualization and by measuring the peak airspeed along the longitudinal axis of the air–jet to plot air–jet trajectory. Airflow visualization allowed for a qualitative image of the airflow patterns. Peak airspeed was measured along the ceiling and floor by a series of velocity measurements in the vertical direction (i.e., downward from the ceiling or upward from the floor) at increments of 2 to 10 cm. The peak airspeed positions along the ceiling and floor were used to describe the air–jet trajectory, which was used in conjunction with airflow visualization to described airflow patterns. Figure 4 highlights the effectiveness of this technique for strong buoyancy–affected (fig. 4a) and low buoyancy–affected (fig. 4b) airflow.

AIRSPEED AND TEMPERATURE MEASUREMENTS

Airspeed was measured using an omni–directional hot–film anemometer (Model 8470, TSI, Inc.). A portable data acquisition system (Model CR10, Campbell Scientific, Inc.) was used to collect data. The average value over time, at a point, was used for analysis and presentation. Temperature was measured using T–type thermocouples. The sampling

Figure 1. Test chamber layout (one constructed for prototype, one for the scale–model). Manure pit volume shown in figure not used during testing.

Figure 2. Dimensions (scale = 1:3 and scale = 1:6) of scale–models. Dimensions given as x/y where x is 1:3 model and y is 1:6 model.

Figure 3. Top–view layout of rubber heating panels. Floor area of 1:3 model was 4,429 m² (6,861 in²) with a heat pad area of 2,603 m² (4,032 in²). Floor area of 1:6 model was 1,107 m² (1,715.2 in²) with a heat pad area of 651 m² (1,008 in²). For both cases, heat/floor area was 0.587.
period was fixed at 180 seconds per collection point at 16 Hz to ensure accurate time–averaged results for turbulent airflow (Thorshauge, 1982). This sampling period was much longer than has been used in previous similar studies (Zhang, 1991; Adre and Albright, 1994).

The non–uniform measurement grid ranged from 0.1 L to 0.9 L horizontally at 0.2 L intervals and from the floor to ceiling vertically at 2 cm to 10 cm intervals (fig. 5). The measured points near the ceiling and floor regions were more closely spaced. This information was used to discretize the peak velocity profile, air–jet trajectory, and the velocity distribution in the room. Approximately 65 to 100 points per run were collected for each model. The above measurements were repeated twice for each test condition.

AIR–JET PENETRATION

The air–jet penetration described in this research was defined as the distance from the inlet wall to where the air–jet separated from the ceiling. The air–jet penetration distance was measured using axial velocity measurements at a point located 5 mm beneath the ceiling. The separation of the wall jet was defined when the decay of axial velocity fell below 0.1 m/s. The air–jet penetration distance was determined by moving the anemometer back and forth at least two iterations to ensure accuracy of the measurement.

The inlet airflow rate was initialized at a small rate and increased gradually to avoid the lag phenomenon found for confined airflow (Zhang, 1991). The air–jet penetration distance was also confirmed qualitatively using airflow visualization and measurements of peak airspeed trajectories.

ASSESSING CRITICAL AIRFLOW RATES

Critical airflow rates were determined using results from the air–jet penetration distance tests. Air–jet penetration showed the approximate upper critical airflow rate for fully rotary airflow and the case where the air–jet fell immediately after entry to the ventilated space (lower critical airflow rate). Airflow rates between the lower and upper critical values were chosen to validate similitude of the velocity and temperature fields between scale–model and prototype.

RESULTS AND DISCUSSION

AIR–JET PENETRATION SIMILARITY

Two temperature difference levels ($\Delta T = 10^\circ C$ and $40^\circ C$) were used to measure the air–jet penetration distance at various airflow rates. The test conditions are shown in table 1. The results measured at $\Delta T = 10^\circ C$ and $40^\circ C$ were also compared with the results for isothermal airflow ($\Delta T = 0^\circ C$) (Yu and Hoff, 1999).
Table 1. Experimental conditions for non–isothermal penetration distance measurements.

<table>
<thead>
<tr>
<th>Test</th>
<th>Q, cfm</th>
<th>Ud, m/s</th>
<th>Tg, °C</th>
<th>Tf, °C</th>
<th>ΔT, °C</th>
<th>Re</th>
<th>Rm</th>
<th>Ar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype</td>
<td>Max.</td>
<td>389</td>
<td>5.98</td>
<td>65</td>
<td>25</td>
<td>40</td>
<td>4778</td>
<td>0.1662</td>
</tr>
<tr>
<td>Min.</td>
<td>28</td>
<td>0.44</td>
<td>65</td>
<td>25</td>
<td>40</td>
<td>349</td>
<td>0.0009</td>
<td>0.0811</td>
</tr>
<tr>
<td>Max.</td>
<td>246</td>
<td>3.77</td>
<td>40</td>
<td>30</td>
<td>10</td>
<td>3014</td>
<td>0.0662</td>
<td>0.0003</td>
</tr>
<tr>
<td>Min.</td>
<td>23</td>
<td>0.36</td>
<td>40</td>
<td>30</td>
<td>10</td>
<td>288</td>
<td>0.0006</td>
<td>0.0311</td>
</tr>
<tr>
<td>Scale–model</td>
<td>Max.</td>
<td>88</td>
<td>5.40</td>
<td>65</td>
<td>25</td>
<td>40</td>
<td>349</td>
<td>0.0009</td>
</tr>
<tr>
<td>Min.</td>
<td>6</td>
<td>0.34</td>
<td>65</td>
<td>25</td>
<td>40</td>
<td>135</td>
<td>0.0005</td>
<td>0.0811</td>
</tr>
<tr>
<td>Max.</td>
<td>55</td>
<td>3.40</td>
<td>40</td>
<td>30</td>
<td>10</td>
<td>1357</td>
<td>0.0536</td>
<td>0.0002</td>
</tr>
<tr>
<td>Min.</td>
<td>5</td>
<td>0.30</td>
<td>40</td>
<td>30</td>
<td>10</td>
<td>121</td>
<td>0.0004</td>
<td>0.0221</td>
</tr>
</tbody>
</table>

The air–jet penetration results for the scale–model and prototype are given as a function of Re, Rm, and Ar (fig. 6). The air–jet penetration distance (Lp/L) increased with increasing Re and Rm and decreased with increasing Ar, as expected. The penetration distance remained constant when the airflow rate reached a threshold value at each specific temperature difference with Re or Rm (fig. 6a,b). The airflow pattern became fully rotary when the penetration distance reached the threshold value. The threshold values for each similitude parameter differed as the temperature difference changed, with a larger temperature difference resulting in a larger threshold value. The differences between ΔT = 10°C and ΔT = 0°C (i.e., isothermal airflow) were not significant, however.

The plots of air–jet penetration distance against Re showed different curves for each enclosure even as the temperature difference remained unchanged. The air–jet penetration distance for the scale–model and prototype fell along the same curve as a function of Rm, provided the temperature difference was similar. However, different curves existed when the temperature difference changed. All data showed very small differences between ΔT = 10°C and ΔT = 0°C.

Figure 6c shows the air–jet penetration distance as a function of Ar. Non–dimensional air–jet penetration distance (Lp/L) within intermediate Ar ranges (0.005 < Ar < 0.015) showed a large difference between scale–model and prototype. This intermediate range was close to the unstable range described by Randall and Battams (1979) (30 < Ar < 75), which corresponds to a range of 0.005 < Ar < 0.012. All non–dimensional air–jet penetration distance data were similar between scale–model and prototype when Ar was at either extreme (Ar > 0.015) or (Ar < 0.005).

It was concluded that the performance for a small temperature difference for non–isothermal airflow (for example: ΔT = 10°C) was similar to isothermal airflow. The maximum non–dimensional air–jet penetration distance for non–isothermal airflow was 0.85 L equaling that for isothermal airflow.

Based on air–jet penetration distance, Re was not found to be the appropriate similitude criterion for non–isothermal airflow. Rm can be used as a similarity requirement for non–isothermal airflow as long as a similar temperature difference is maintained between the scale–model and prototype. Ar can be treated as the similitude criterion only at the regions of extreme Ar values of (i.e., Ar < 0.005 or Ar > 0.015). The threshold (or critical) value of Re and Rm to reach 98% of the threshold penetration distance for a temperature difference of ΔT = 0°C, 10°C, and 40°C for both the scale–model and prototype are shown in table 2. The equation describing air–jet penetration distance as a function...
of Re and Rm are also shown in table 2. Isothermal results from Yu and Hoff (1999) are included for comparison purposes.

AIRFLOW PATTERN SIMILARITY

Air–jet trajectory comparisons were used to quantitatively verify airflow patterns between scale–model and prototype under non–isothermal conditions. The comparisons are presented based on either Ar or Rm for ∆T = 40°C (fig. 7). The test conditions for quantifying air–jet trajectory are given in table 3 and are divided into eleven prototype (NPx) and twelve scale–model (NMx) experiments.

Air–jet trajectories were compared and the similitude parameter that showed the better consistency between measured data from the scale–model and prototype was chosen as the appropriate similitude parameter. Based on the results shown in figure 7, it was concluded that using Ar as the similitude criterion gave better consistency between data with a similar temperature difference when the airflow pattern within the enclosure consisted of two circulation airflow zones. These cases are shown in figures 7 (a, c, e, and g). When the airflow rate increased to produce a single circulation airflow zone, using Rm with a similar temperature difference gave better consistency of the measured data as shown in figures 7 (b, d, f, and h). When the airflow rate was beyond the critical condition where the air–jet remained horizontal, there was no significant difference between the air–jet trajectories using either Ar or Rm similarity. This was due to the fact that the airflow pattern is independent of the airflow rate when it is beyond the threshold value.

The air–jet trajectory results showed that the Ar might be used as the similitude criterion as long as the buoyancy force dominates the airflow pattern for low inlet jet momentum conditions (i.e., low inlet velocity). When the airflow rate increases, the air–jet behaves like isothermal airflow and the similitude criteria changes to the Rm assuming similar temperature differences between the scale–model and prototype.

The critical values measured and used to distinguish the behavior of non–isothermal air–jets are summarized in figure 8 and table 4. Figure 8 was generated based on observed (airflow visualization) and measured (air–jet trajectory) airflow patterns. At the lower critical Ar< 0.005, a single–circulation airflow pattern exists and the airflow pattern behaves like isothermal flow, where either Ar or Rm may be used as the similitude parameter. At the upper critical Ar> 0.015, buoyancy forces dominate and Ar should be used as the similitude parameter. For intermediate ranges (0.005< Ar < 0.015), the better similitude parameter for duplicating the airflow pattern is Rm with the same temperature difference.

OCCUPIED ZONE AIRSPEED AND TEMPERATURE SIMILARITY

The occupied zone maximum airspeed and corresponding dimensionless temperature results are summarized for Rm similarity (fig. 9a,b) and Ar similarity (fig. 9c) as a function of axial distance from the inlet diffuser. Figure 9 represents the summary regression results from all tests conducted (table 5). For cases where Ar was the similarity criteria (Ar>0.015), little axial variation in occupied zone airspeed and dimensionless temperature existed and thus these results were grouped for all axial locations between 0.3 and 0.7L.
Prototype (top photo): $Q_p = 53$ cfm; $\Delta T = 40^\circ$C; $\text{Ar}_p = 0.0239$; $\text{Rm}_p = 0.003$.
Model (bottom photo): $Q_m = 9$ cfm; $\Delta T = 40^\circ$C; $\text{Ar}_m = 0.0270$; $\text{Rm}_m = 0.001$.

Figure 7 (part). Air–jet trajectory comparison at the ceiling and near–floor regions based on either $\text{Ar}$ (a, c, e, g) or $\text{Rm}$ (b, d, f, h) as the similarity criteria (continued next pages).

**PRACTICAL USE OF THE RESULTS**

Using scale–models to quantify performance of full–scale prototypes is not an easy task. The results from this research are summarized in this section as a proposed procedure for doing scale–model studies of non–isothermal behavior in ceiling slot–ventilated livestock ventilation systems.

**SCALE–MODEL DESIGN PROCEDURE**

Scale–model design for investigating non–isothermal behavior begins with a knowledge of the prototype conditions of interest. The procedure proposed begins with a description of the Archimedes Number (eq. 2) using parameters specific to a slot–ventilated system:
Figure 7 (part). Air–jet trajectory comparison at the ceiling and near–floor regions based on either Ar (a, c, e, g) or Rm (b, d, f, h) as the similarity criteria (continued next pages).

\[
\begin{align*}
Q_p &= 72 \text{ cfm}; \ \Delta T = 40^\circ\text{C}; \ Ar_p = 0.0130; \ Rm_p = 0.006; \ Q_m = 14 \text{ cfm}; \ \Delta T = 40^\circ\text{C}; \ Ar_m = 0.0113; \ Rm_m = 0.003.
\end{align*}
\]

Given in table 6 is the expected airflow pattern duplicated in the scale–model relative to the prototype conditions. Case 1, representing strong buoyancy–affected airflow, corresponds to Randall and Battams (1979) condition of ArL > 75.

**Predicting Prototype Behavior Based on Scale–Model Experimental Data**

Several key thermal environmental factors measured in the scale–model can be determined and used for predicting
prototype behavior. The air–jet penetration distance, or maximum “throw” of the air–jet in the prototype, can be predicted using the regression relations given in table 2. Additionally, the maximum occupied zone airflow and corresponding temperature levels can be predicted using the regression relations given in table 5.

**A Practical Design Problem**

Assume the full–scale prototype shown in figure 10 is studied experimentally with a scale–model of \( n = 5 \) (\( L_p/L_m \)). If the prototype is ventilated at 8 ACH (\( = 0.8 \text{ m}^3/\text{s} \)) with a continuous slot width of 1 cm (\( U_d = 4 \text{ m/s} \)) allowing 0°C air to enter, the analysis for setting a scale–model would be as follows:

**Step 1. Determine \( Ar_p \)**

\[
Ar_p = \frac{(9.8)(2/(546+33+0))(33)(0.01)}{(0.8/(20*0.01))^2}
\]

\[ Ar_p = 0.0007 \text{ (assumes 33°C floor temperature)} \]
Table 3. Experimental conditions for quantifying non-isothermal air-jet trajectory measurements.

<table>
<thead>
<tr>
<th>Test</th>
<th>Q, cfm</th>
<th>(U_d, \text{m/s})</th>
<th>(T_d, ^\circ\text{C})</th>
<th>(T_c, ^\circ\text{C})</th>
<th>(\Delta T, ^\circ\text{C})</th>
<th>Re</th>
<th>Rm</th>
<th>Ar</th>
<th>Arc</th>
</tr>
</thead>
<tbody>
<tr>
<td>NP1</td>
<td>324</td>
<td>4.97</td>
<td>61.5</td>
<td>21.5</td>
<td>40</td>
<td>3976</td>
<td>0.115</td>
<td>0.0006</td>
<td>4</td>
</tr>
<tr>
<td>NP2</td>
<td>221</td>
<td>3.39</td>
<td>61.5</td>
<td>21.5</td>
<td>40</td>
<td>2708</td>
<td>0.053</td>
<td>0.0014</td>
<td>8</td>
</tr>
<tr>
<td>NP3</td>
<td>150</td>
<td>2.30</td>
<td>57</td>
<td>17</td>
<td>40</td>
<td>1842</td>
<td>0.025</td>
<td>0.0030</td>
<td>18</td>
</tr>
<tr>
<td>NP4</td>
<td>105</td>
<td>1.61</td>
<td>57.5</td>
<td>17.5</td>
<td>40</td>
<td>1284</td>
<td>0.012</td>
<td>0.0062</td>
<td>37</td>
</tr>
<tr>
<td>NP5</td>
<td>72</td>
<td>1.10</td>
<td>54.5</td>
<td>14.5</td>
<td>40</td>
<td>882</td>
<td>0.006</td>
<td>0.0133</td>
<td>80</td>
</tr>
<tr>
<td>NP6</td>
<td>58</td>
<td>0.90</td>
<td>55.5</td>
<td>15.5</td>
<td>40</td>
<td>716</td>
<td>0.004</td>
<td>0.0201</td>
<td>121</td>
</tr>
<tr>
<td>NP7</td>
<td>53</td>
<td>0.81</td>
<td>62.77</td>
<td>22.77</td>
<td>40</td>
<td>649</td>
<td>0.003</td>
<td>0.0239</td>
<td>144</td>
</tr>
<tr>
<td>NP8</td>
<td>116</td>
<td>1.78</td>
<td>82.33</td>
<td>22.33</td>
<td>60</td>
<td>1424</td>
<td>0.015</td>
<td>0.0072</td>
<td>44</td>
</tr>
<tr>
<td>NP9</td>
<td>83</td>
<td>1.28</td>
<td>79.37</td>
<td>19.37</td>
<td>60</td>
<td>1025</td>
<td>0.008</td>
<td>0.0141</td>
<td>85</td>
</tr>
<tr>
<td>NP10</td>
<td>73</td>
<td>1.12</td>
<td>81.94</td>
<td>21.94</td>
<td>60</td>
<td>898</td>
<td>0.006</td>
<td>0.0182</td>
<td>110</td>
</tr>
<tr>
<td>NP11</td>
<td>59</td>
<td>0.90</td>
<td>79.56</td>
<td>19.56</td>
<td>60</td>
<td>723</td>
<td>0.004</td>
<td>0.0283</td>
<td>170</td>
</tr>
<tr>
<td>NM1</td>
<td>54</td>
<td>3.34</td>
<td>61</td>
<td>21</td>
<td>40</td>
<td>1334</td>
<td>0.052</td>
<td>0.0007</td>
<td>4</td>
</tr>
<tr>
<td>NM2</td>
<td>38</td>
<td>2.36</td>
<td>60</td>
<td>20</td>
<td>40</td>
<td>942</td>
<td>0.026</td>
<td>0.0014</td>
<td>9</td>
</tr>
<tr>
<td>NM3</td>
<td>26</td>
<td>1.60</td>
<td>60</td>
<td>20</td>
<td>40</td>
<td>641</td>
<td>0.012</td>
<td>0.0031</td>
<td>19</td>
</tr>
<tr>
<td>NM4</td>
<td>18</td>
<td>1.08</td>
<td>60</td>
<td>20</td>
<td>40</td>
<td>433</td>
<td>0.005</td>
<td>0.0068</td>
<td>41</td>
</tr>
<tr>
<td>NM5</td>
<td>14</td>
<td>0.84</td>
<td>62.8</td>
<td>22.8</td>
<td>40</td>
<td>334</td>
<td>0.003</td>
<td>0.0113</td>
<td>68</td>
</tr>
<tr>
<td>NM6</td>
<td>11</td>
<td>0.70</td>
<td>60</td>
<td>20</td>
<td>40</td>
<td>281</td>
<td>0.002</td>
<td>0.0161</td>
<td>97</td>
</tr>
<tr>
<td>NM7</td>
<td>9</td>
<td>0.54</td>
<td>58</td>
<td>18</td>
<td>40</td>
<td>217</td>
<td>0.001</td>
<td>0.0270</td>
<td>163</td>
</tr>
<tr>
<td>NM8</td>
<td>28</td>
<td>1.72</td>
<td>82.79</td>
<td>22.79</td>
<td>60</td>
<td>687</td>
<td>0.014</td>
<td>0.0039</td>
<td>23</td>
</tr>
<tr>
<td>NM9</td>
<td>21</td>
<td>1.28</td>
<td>79.83</td>
<td>19.83</td>
<td>60</td>
<td>513</td>
<td>0.008</td>
<td>0.0070</td>
<td>42</td>
</tr>
<tr>
<td>NM10</td>
<td>16</td>
<td>0.96</td>
<td>79.8</td>
<td>19.8</td>
<td>60</td>
<td>382</td>
<td>0.004</td>
<td>0.0127</td>
<td>76</td>
</tr>
<tr>
<td>NM11</td>
<td>14</td>
<td>0.83</td>
<td>81.71</td>
<td>21.71</td>
<td>60</td>
<td>333</td>
<td>0.003</td>
<td>0.0165</td>
<td>100</td>
</tr>
<tr>
<td>NM12</td>
<td>11</td>
<td>0.68</td>
<td>82.33</td>
<td>22.33</td>
<td>60</td>
<td>273</td>
<td>0.002</td>
<td>0.0246</td>
<td>148</td>
</tr>
</tbody>
</table>

\[\text{[a]} \text{N = nonisothermal, P = prototype, M = scale-model, 1 = test 1.}\]

Table 4. The critical values of non-isothermal airflow patterns determined using air-jet trajectories.

<table>
<thead>
<tr>
<th>Classification of airflow pattern</th>
<th>Critical values measured</th>
<th>Similitude criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air–jet falls on entry or two–circulation airflow</td>
<td>(\text{Ar} &gt; 0.015)(\text{Ar}_c &gt; 90)</td>
<td>Archimedes number</td>
</tr>
<tr>
<td>Single–circulation airflow, air–jet falls between inlet and end wall</td>
<td>(0.005 &lt; \text{Ar} &lt; 0.015)(30 &lt; \text{Ar}_c &lt; 90)</td>
<td>Rm with same heat load between model and prototype</td>
</tr>
<tr>
<td>Air–jet remains horizontal, and behaves as isothermal airflow</td>
<td>(\text{Ar} &lt; 0.005)(\text{Ar}_c &lt; 30)</td>
<td>Either (\text{Ar}) or (\text{Rm}) may be used</td>
</tr>
</tbody>
</table>

**Step 2. Determine flow regime criteria**

Since \(\text{Ar}_p < 0.005\), the air jet behaves as isothermal airflow and the Momentum Ratio, \(\text{Rm}\), with similar \((T_f - T_d)\) is the appropriate scaling criteria (table 4).

**Step 3. Determine \(\text{Rm}\) scaling criteria**

\[\text{Rm}_p = \text{Rm}_m = (0.01)(4)^2/(6+3) = 0.018\]

with \((T_f - T_d)_m = (T_f - T_d)_p = 33–0 = 33^\circ\text{C}\.\]
Figure 8. Critical values to distinguish non–isothermal airflow patterns based on normalized air–jet penetration distance versus Archimedes number.

Figure 9. Summary regression equations describing the occupied zone (a) airspeed and (b) dimensionless temperature levels as a function of Momentum Ratio (Rm) and axial distance for non–isothermal conditions where Ar<0.015, and, (c) as a function of Archimedes Number (Ar) for non–isothermal cases where Ar > 0.015. Dimensionless temperature defined as \((T_{max} - T_d) / (T_f - T_d)\).

Table 5. The regression equations of peak floor airspeed at different positions of both scale–model and prototype versus Rm or Ar.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Positions</th>
<th>Regression equations</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non–Isothermal</td>
<td>Ar &lt; 0.015</td>
<td>0.3L (U_{rm} = 3.44\sqrt{R_m})</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>0.5L</td>
<td>(U_{rm} = 4.53\sqrt{R_m})</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>0.7L</td>
<td>(U_{rm} = 4.96\sqrt{R_m})</td>
<td>0.99</td>
</tr>
<tr>
<td>Ar &gt; 0.015</td>
<td>0.3L to 0.7L</td>
<td>(U_{rm} = 0.03/\text{Ar})</td>
<td>0.77</td>
</tr>
<tr>
<td>Isothermal</td>
<td>0.3L</td>
<td>(U_{rm} = 2.65\sqrt{R_m})</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>0.5L</td>
<td>(U_{rm} = 3.29\sqrt{R_m})</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>0.7L</td>
<td>(U_{rm} = 3.52\sqrt{R_m})</td>
<td>0.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Positions</th>
<th>Regression equations</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar &lt; 0.015</td>
<td>0.3L</td>
<td>(\frac{T_{max} - T_d}{T_f - T_d} = 0.086R_m^{-0.30})</td>
<td>0.83</td>
</tr>
<tr>
<td>Ar &lt; 0.015</td>
<td>0.5L</td>
<td>(\frac{T_{max} - T_d}{T_f - T_d} = 0.066R_m^{-0.35})</td>
<td>0.86</td>
</tr>
<tr>
<td>Ar &gt; 0.015</td>
<td>0.7L</td>
<td>(\frac{T_{max} - T_d}{T_f - T_d} = 0.054R_m^{-0.38})</td>
<td>0.88</td>
</tr>
<tr>
<td>Ar &gt; 0.015</td>
<td>0.3L to 0.7L</td>
<td>(\frac{T_{max} - T_d}{T_f - T_d} = 0.1\ln(\text{Ar}) + 0.87)</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Treat \(\Delta T = 0\,^\circ\text{C}\) as \(\Delta T = 10\,^\circ\text{C}\) in summer time, and the temperature environment is similar as the isothermal condition.

Step 4. Select scaling factor and determine scale–model airflow rate

Let \(n = 5\), yielding a geometrically scaled model of \((20/5) \times (6/5) \times (3/5)\) m and an airflow rate delivery of (table 6)
Table 6. Airflow patterns and distinguishing criteria.

<table>
<thead>
<tr>
<th>Airflow patterns</th>
<th>Critical values</th>
<th>Similitude criteria</th>
<th>Critical airflow rate, $Q_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two–circulation</td>
<td>$Ar &gt; 0.015$</td>
<td>$Ar$</td>
<td>$= Q_p/n^2$</td>
</tr>
<tr>
<td>Single–circulation</td>
<td>$0.015 &gt; Ar &gt; 0.005$</td>
<td>$R_m, \Delta T$</td>
<td>$= Q_p n^{-2}$</td>
</tr>
<tr>
<td>Fully rotary</td>
<td>$Ar &lt; 0.005$</td>
<td>$R_m, \Delta T$</td>
<td>$= Q_p n^{-2}$</td>
</tr>
</tbody>
</table>

[a] $R_m$ with $(T_f-T_d)$ equivalent between prototype and scale–model.

$$Q_m = \frac{Q_p}{n^2} = 0.8/5^2 = 0.032 \text{ m}^3/\text{s}$$

**Step 5. Determine scale–model slot height**

$$h_m = \frac{0.032/(20/5)^2}{0.018 (6/5 +3/5)} = 0.0020 \text{ m}$$

**Step 6. Expected scale–model and thus prototype behavior**

a. Airflow pattern (from table 6): Full rotary airflow
   - $L_p/L = 0.84 /\{20.18 \exp(-47.29*\sqrt{0.018}) + 1\} = 0.81$

b. Axial distribution of occupied zone airspeed (from table 5):
   - $U_{rm} = 3.44 \sqrt{0.018} = 0.46 \text{ m/s} @ x/L = 0.30$
   - $U_{rm} = 4.53 \sqrt{0.018} = 0.61 \text{ m/s} @ x/L = 0.50$
   - $U_{rm} = 4.96 \sqrt{0.018} = 0.67 \text{ m/s} @ x/L = 0.70$

c. Axial distribution of occupied zone temperature (from table 5, without supplemental heat):
   - $T_{max} = 33 (0.086/0.018^{0.30} + 0^\circ C = 9.5^\circ C @ x/L = 0.30$
   - $T_{max} = 33 (0.066/0.018^{0.35} + 0^\circ C = 8.9^\circ C @ x/L = 0.50$
   - $T_{max} = 33 (0.054/0.018^{0.38} + 0^\circ C = 8.2^\circ C @ x/L = 0.70$

d. Axial distribution of occupied zone temperature (from table 5, with supplemental heat):
   - $T_{max} = 33 (0.086/0.018^{0.30} + 0^\circ C = 9.5^\circ C @ x/L = 0.30$
   - $T_{max} = 33 (0.066/0.018^{0.35} + 0^\circ C = 8.9^\circ C @ x/L = 0.50$
   - $T_{max} = 33 (0.054/0.018^{0.38} + 0^\circ C = 8.2^\circ C @ x/L = 0.70$

**CONCLUSIONS**

Scale–model studies of the ventilation characteristics inside enclosures is a reliable method to simulate airflow patterns and occupied zone airspeed and temperature in a prototype provided that an acceptable similitude scaling parameter can be found.

The comparison of using $Ar$ and $R_m$ with the same temperature difference between the scale–model and prototype for non–isothermal airflow was conducted using measurements of air–jet penetration distance, airflow pattern, and airspeed/temperature distributions. From these results the following conclusions were made:

- The threshold penetration distance was 0.85 L, where the threshold $R_m$ was 0.066 for $\Delta T = 40^\circ C$ and 0.02 for $\Delta T = 10^\circ C$ for both the scale–model and prototype.
- The penetration distance was found to be a function of the $R_m$ and is described in table 2.
- The non–isothermal airflow patterns were classified as shown in table 6, with ranges similar to the results shown by previous research (Mullejans, 1966; Randall and Battams, 1979).
- Fully rotary airflow patterns were self–similar, and since the airspeed field affected the thermal environment, it was shown that similitude could be reached with a similar $R_m$ and temperature difference.
- Non–isothermal airflow will be similar to isothermal airflow when the $Ar$ is below 0.005 ($Ar_c = 30$), which agrees with Randall and Battams (1979).
- $Ar$ is the appropriate similitude criterion between scale–model and prototype when the airflow pattern is a two–circulation zone airflow, or when $Ar > 0.015$. When $Ar$ is below 0.005 ($Ar_c < 30$), the behavior of non–isothermal airflow is similar to isothermal airflow with $R_m$ and the same temperature difference as the appropriate similarity criteria. Fully rotary airflow was reached with an increased airflow rate or decreased temperature difference. For either of these conditions, $R_m$ with the same temperature difference between scale–model and prototype was found to be the appropriate similarity criteria.

**REFERENCES**


**NOMENCLATURE**

**SYMBOLS**

\[
\begin{align*}
\text{Ar} &= \text{Archimedes number defined as } \frac{gh(T_f - T_d)}{U_d^2} \\
\text{Ar}_c &= \text{corrected Archimedes number defined as } \\
\frac{CdghWHW + H(T_w - T_d)}{(546 + T_w + T_d)Q^2} \\
\text{b} &= \text{diffuser length (m)} \\
\text{Cd} &= \text{discharge coefficient} \\
\text{Eu} &= \text{Euler number defined as } \frac{2\Delta P}{\rho U^2} \\
\text{Fr} &= \text{Froude number defined as } \frac{U}{\sqrt{gL}} \\
\text{g} &= \text{gravitational acceleration rate (m/s}^2) \\
\text{h} &= \text{diffuser opening height (m)} \\
\text{H} &= \text{room height (m)} \\
\text{L} &= \text{room length (m)} \\
\text{Lp} &= \text{penetration distance defined by Adre and Albright (1994)} \\
\text{n} &= \text{geometry scale ratio of prototype to model} \\
\text{P} &= \text{thermodynamic pressure (Pa)} \\
\text{Pe} &= \text{Peclet number defined as } \frac{UL}{\alpha} \\
\text{Pr} &= \text{Prandtl number defined as } \frac{\alpha}{\nu} \\
\text{q} &= \text{heat transfer rate (W)} \\
\text{Q} &= \text{ventilation rate (m}^3\text{/s unless otherwise noted)} \\
\text{Re} &= \text{Reynolds number defined as } \frac{hU_d^2}{\nu} \\
\text{Rm} &= \text{inlet jet momentum ratio defined as } \frac{hU_d^2}{L + H} \\
\text{T} &= \text{mean temperature and fluctuation component (°C)} \\
\text{ΔT} &= \text{mean temperature and fluctuation component (°C)} \\
\text{U} &= \text{mean air velocity (m/s)} \\
\text{V} &= \text{room volume (m}^3) \\
\text{W} &= \text{room depth (m)} \\
\end{align*}
\]

**GREEK SYMBOLS**

\[
\begin{align*}
\alpha &= \text{thermal diffusion coefficient, (m}^2\text{/s)} \\
\beta &= \text{thermal expansion coefficient defined as } \frac{1}{(T_f + T_d)/2}, (1/K) \\
\nu &= \text{kinematic viscosity, (m}^2\text{/s)} \\
\mu &= \text{dynamic viscosity (Ns/m}^2) \\
\rho &= \text{density of air, (kg/m}^3) \\
\end{align*}
\]

**SUBSCRIPTS**

\[
\begin{align*}
c &= \text{characteristic scales} \\
d &= \text{diffuser} \\
f &= \text{floor} \\
m &= \text{model} \\
o &= \text{outside of room} \\
p &= \text{prototype} \\
w &= \text{wall} \\
\end{align*}
\]