Circular Motion

Suppose that an object moves around a circle of radius \( r \) at a constant speed. Then the speed \( v \) of the object, sometimes called linear speed, is the total distance traveled \( s \) divided by the elapsed time \( t \).

\[
(1) \quad v = \frac{s}{t}
\]

Now let \( \theta \) be the angle swept out in time \( t \) as the object moves around the circle. Then the angular speed \( \omega \) of the object is defined to be the angle \( \theta \), measured in radians, divided by the elapsed time \( t \).

\[
(2) \quad \omega = \frac{\theta}{t}
\]

Angular speed is often described in terms of revolutions per unit time, such as 600 rpm (revolutions per minute). Since 1 revolution = \( 2\pi \) radians, the angular speed of such an object is

\[
600 \text{ revolutions/minute} = 600 \text{ revolutions/minute} \cdot \frac{2\pi \text{ radians}}{\text{revolution}} = 1200\pi \text{ radians/minute}
\]

If the object moves along an arc of length \( s \), then the relationship \( s = r\theta \) between arc length and radian measure (given in section 5.3) yields a corresponding relationship between linear and angular speed:

\[
(3) \quad v = \frac{s}{t} = \frac{\theta}{t} = r\omega
\]

Linear speed has the dimension of length per unit of time, such as feet per second or miles per hour. Since radians are unitless and \( r \) has the dimension of length, notice that the right side of equation (3) also has the correct dimension of length per unit of time.
Circular Motion - Examples

1. Suppose the wheels on your bicycle are 28 inches in diameter, and you are traveling at a speed of 10 miles per hour. How many revolutions per minute are the wheels turning?

   Solution: First convert speed to inches per minute:
   \[
   \nu = 10 \text{ miles/hour} \cdot \frac{5280 \text{ feet}}{\text{mile}} \cdot \frac{12 \text{ inches}}{\text{foot}} = 633,600 \text{ inches/hour}
   \]
   \[
   = \frac{633,600 \text{ inches}}{\text{hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minute}} 
   \]
   \[
   \approx 10,660 \text{ inches/minute}
   \]
   Now the radius of each wheel is \( r = 14 \) inches, so \( \nu = r \omega \) from equation (3) implies
   \[
   \omega = \frac{\nu}{r} \approx \frac{10,580}{14} \text{ radians/minute} \approx 754.3 \text{ radians/minute}
   \]
   Finally, converting to revolutions per minute yields
   \[
   \frac{\text{754.3 radians}}{\text{minute}} \cdot \frac{1 \text{ revolution}}{2\pi \text{ radians}} \approx 120 \text{ revolutions/minute}
   \]

2. Suppose that you are competing in the hammer throw, which consists of spinning a weight on the end of a rope and then releasing it. If the rope is 2.5 feet long and you spin the weight at a rate of 120 revolutions/minute, how fast does the weight travel when it is released?

   Solution: First convert to radians per minute:
   \[
   \frac{120 \text{ revolutions}}{\text{minute}} \cdot \frac{2\pi \text{ radians}}{\text{revolution}} = 240\pi \text{ radians/minute} = \omega
   \]
   Now using equation (3),
   \[
   \nu = r \omega = 2.5 \text{ feet} \cdot 240\pi \text{ radians/minute} \approx 1386 \text{ feet/minute}
   \]
   or equivalently, \( \approx 21.4 \text{ miles/hour} \).
Circular Motion Exercises

1. How many revolutions per minute are required to produce a hammer throw traveling at 30 miles per hour? As in example (2), assume that the rope is 2.5 feet long.

Solution: First convert speed to feet per minute:
\[ v = \frac{30 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2640 \text{ feet/minute}. \]

Now \( r = 2.5 \text{ feet} \), so \( v = r\omega \) implies \( \omega = \frac{v}{r} \)
\[ = \frac{2640}{2.5} \text{ radians/minute} = 1056 \text{ radians/minute}. \]

Finally, converting to revolutions per minute yields
\[ 1056/(2\pi) \text{ revolutions/minute} \approx 168 \text{ revolutions/minute}. \]

2. Suppose the wheels on your bicycle are 26 inches in diameter, and the wheels are turning at 4 revolutions per second. How fast are you traveling?

Solution: First find the angular speed in radians per minute:
\[ 4 \text{ revolutions/second} = 4 \times 2\pi \times 60 \text{ radians/minute} \]
\[ = 480\pi \text{ radians/minute} = \omega. \]

Now \( v = r\omega = 13 \text{ inches} \times 480\pi \text{ inches/minute} \approx 19,604 \text{ inches/minute}, \)
or equivalently, \( \approx 18.56 \text{ miles/hour}. \)

3. The cable on a chair lift at a ski resort is driven by a 6 foot diameter pulley that rotates at a speed of 30 revolutions per minute. How fast is each chair on the chair lift moving?

Solution: First find the angular speed in radians per minute:
\[ 30 \text{ revolutions/minute} = 30 \times 2\pi \text{ radians/minute} \]
\[ = 60\pi \text{ radians/minute} = \omega. \]

Now \( v = r\omega = 3 \text{ feet} \times 60\pi \text{ feet/minute} \approx 565.5 \text{ feet/minute}, \)
or equivalently, \( \approx 6.4 \text{ miles/hour}. \)

4. Suppose the wheels on your car are 16 inches in diameter, and you are traveling at a speed of 60 miles per hour. How many revolutions per minute are the wheels turning?

Solution: First convert speed to inches per minute:
\[ v = \frac{60 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 63,360 \text{ inches/minute}. \]

Now \( r = 8 \text{ inches} \), so \( v = r\omega \) implies \( \omega = \frac{v}{r} \)
\[ = \frac{63,360}{8} \text{ radians/minute} = 7920 \text{ radians/minute}. \]

Finally, converting to revolutions per minute yields
\[ 7920/(2\pi) \text{ revolutions/minute} \approx 1260.5 \text{ revolutions/minute}. \]